On-line Voltage Stability Load Index Estimation Based on PMU Measurements

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Abstract— A new method for estimation of Voltage Stability Load Index (VSLI) for a power system using data from optimally located phasor measurement units (PMU) is presented in this paper. Optimal PMU placement was carried out considering islanding operating conditions. VSLI is estimated using a kind of recurrent neural network know as the Echo State Network (ESN). The development ESN is computational efficient and provides accurate estimation. Results of optimal PMU placement for normal and islanded operating conditions, and ESN estimation of VSLI on the IEEE 14 bus system is presented.

***Index Terms*— Echo state networks, PMUs, power system islanding, voltage stability, voltage stability load index**

I. Introduction

POWER system voltage stability has become an issue of great concern for both power system planning and operation in recent years, as a result of a number of major black outs that have been experienced in many countries due to voltage stability problems [1,2]. This has been mainly due to power systems being operated closer to their stability limits because of increased demand for electricity [1]. Many studies have been carried out to determine voltage stability indices in order to facilitate necessary control actions to preclude eminent instability and thereby improving voltage stability in a power system. References [3, 4] present comparative studies and analysis of six different voltage stability indices, while [5] introduces the voltage stability load bus index (VSLI). In order to obtain VSLI analytically, no-load voltage phasor information is required at each bus for a given system topology. Since no-load voltage is dependent upon the system topology and operating point, it varies as the system topology or operating point changes. In practice it is difficult to obtain no-load voltage phasor at a bus each time the system topology or operating point changes. Reference [6] introduces a method of using an artificial neural network to predict the voltage stability margin of a power system obtained by continuation power flow based on synchrophasor measurements of voltage magnitudes and phase angles. The voltage stability margin gives the distance of the power system to the critical load at which the system becomes voltage unstable in a specified load direction. However for local monitoring it is helpful to have voltage stability index at individual load buses in the system [7]. A technique that uses local information at every load bus to obtain the Voltage Stability Load Index (VSLI) as a measure of the distance to instability at the bus has been presented in [7]. However, this technique requires knowledge of the two bus equivalent system at every bus to calculate the voltage stability load bus index.

A new method to estimate the VSLI at each load bus based on synchrophasor measurements of voltage magnitudes and angles at load buses. The estimation of the VSLI is carried out by an Echo State Network (ESN). A real-time model of the IEEE 14-bus test system on the Real-Time Digital Simulator (RTDS) has been used to investigate the ESN based approach. Phasor Measurement Units (PMUs) are placed at pre-determined buses that ensure that the system is fully observable for voltage stability monitoring. Placement of PMUs for voltage stability monitoring is done is such a way as to ensure that voltage phasors at all load buses are either direct measurements form PMUs or calculated at first level of observability. The method for optimal PMU placement developed in this study ensures that the power system is fully observable during normal operation and under islanded conditions. Reference [8] presents a technique for splitting the system into smaller islands followed by load shedding for the purpose of preventing wide spread system failure and black out. During these instances monitoring and control of voltage stability remain critical for the system to survive the disturbance.

II. Voltage Stability load Index

The voltage stability load index used in this paper is calculated from PMU measurements of voltage magnitudes and angles at load buses. PMUs can provide real-time measurements of voltage phasors and incident current phasors. This information can be adequately used to detect voltage stability margin directly from the measurements and in real-time [10]. The minimum number of PMUs that make the system observable are placed at pre-determined buses to take direct measurements while voltage phasor information at the remaining buses is calculated from these measurements and known system transmission line impedances.

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The mathematical formulation of VSLI used in this study is presented in [5]. The voltage stability load index used is derived from voltage equations of the Thevenin equivalent representation of the system at the load bus (Fig.1).

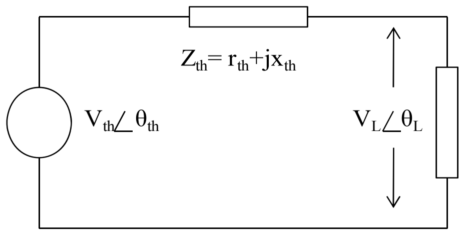

Fig.1. Thevenin equivalent representation at a load bus

The voltage stability load index in terms of voltage phasors at the load bus can be derived as the following [5].

$$VSLI = \frac{4\left|V_0 V_L \cos(\theta_0 - \theta_L) - V_L^{\,2} \cos^2(\theta_0 - \theta_L)\right|}{V_0^{\,2}} \tag{1}$$

Where V_0 and V_L are the no-load and load voltage magnitudes at the load bus, respectively, and θ_0 and θ_L are the no-load and load voltage phase angle measurements respectively. The value of voltage stability load index varies from zero at no-load to one at the point of voltage stability limit or collapse. The point of voltage stability limit corresponding to VSLI value of one is the point at which the load factor is maximum (Fig. 2).

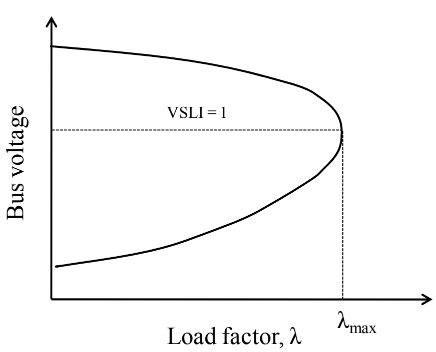

Fig. 2. P-V curve at a load bus showing VSLI. Of one at the maximum load factor.

Analysis of voltage stability using P-V curves as shown in Fig. 2 requires solution of the power flow equations of the system that may be computationally intensive and thus not suitable for on-line applications.

III. Power System Observability

Optimal placement of PMUs in power systems is done in a manner that ensures that all system bus voltages can be observed using the minimum possible number of PMUs. This is important from the economic and technical point of views. PMUs are used in power system for system monitoring, and state estimation that can facilitate on-line control and protection operations. However, during contingencies, the system may not be fully observable for example when measurements from one PMU are lost and there is not enough redundancy. Further, during emergencies, a power system may be operated in an islanded mode. In this case the system is separated into smaller islands at reduced load in order to avoid system wide failure or blackout. During islanding conditions optimal placement of PMUs is vital as it is necessary to monitor the separated islands and maintain voltage stability. A method for optimal PMU placement that ensures that the system is fully observable during normal operating conditions and under conditions of islanding using the minimum number of PMUs is presented in this paper. Fig. 3 shows the flowchart for optimal PMU placement using Genetic Algorithm (GA). Besides GA other methods are also applicable such as particle swarm optimization [17].

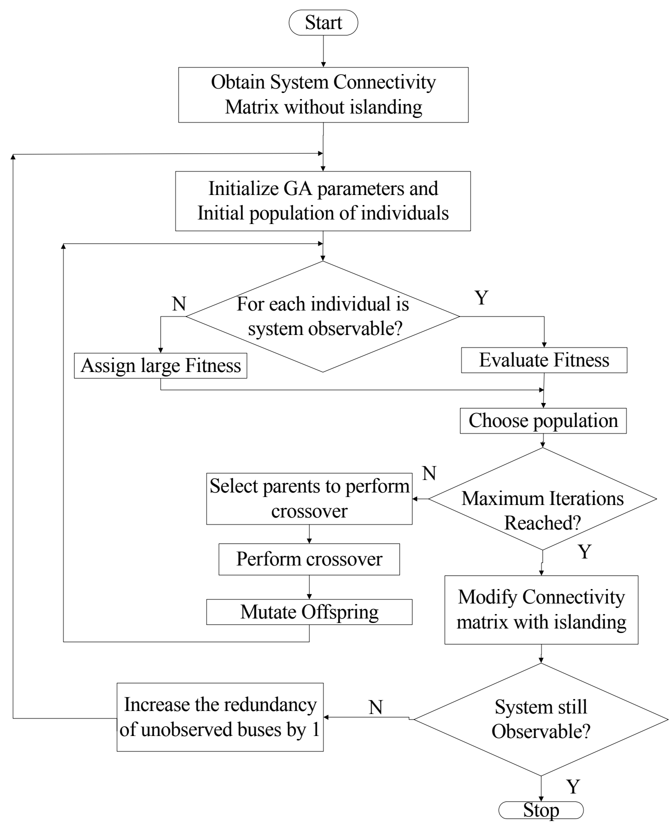

Fig. 3. Flowchart for optimal PMU placement consideration islanding using GA.

The procedure for determining optimal PMU locations in a power system considering islanding can be summarized in the following steps:

1. Obtain optimum PMU locations for normal operating conditions using GA.
2. Split system into the desired set of islands and modify the system connectivity matrix.
3. Use solution obtained in step 1 and test if islanded system (the islands are) is still observable.
4. If islanded system is also observable, the solution in step 1 is the required solution.

5. If islanded system is not observable with solution in step 1, identify buses not observed after islanding.
6. Increase redundancy at each bus that becomes unobservable after islanding by one.
7. Repeat steps 1 to 6 until the optimum solution that makes the system observable for both normal and islanded conditions is obtained.

The following rules have been used for optimal PMU placement [11]:

1. Assign a voltage measurement to a bus where a PMU is placed including a current measurement to each line incident at the bus (assuming that the number of channels of each PMU is at least one less than the maximum number of incident lines), the current in the remaining line can be obtained by Kirchoff's current law.
2. Assign a pseudo-voltage measurement to each node reached by a node that has a PMU. The voltage measurement of such a bus is obtained from the known voltage of the bus with a PMU, line current and impedance of the line.
3. Assign a pseudo-current to each line connecting two buses whose voltages are known. The current is such a branch is readily obtained from the two known voltages and line impedance using Ohm's law.
4. Assign a pseudo-current measurement to an incident line where all other lines current measurements are known. The unknown current is obtained using Kirchoff's current law.

Equations for evaluation of fitness function, F used with the GA algorithm to determine the optimal number and location of PMUs in the system are as follows:

$$F = \begin{cases} K \ \ if\, not\, observable \\ w_1 F_1 + w_2 F_2,\ otherwise \end{cases} \tag{2}$$

Where,

$$F_1 = x^T x \tag{3}$$

$$F_2 = (N\text{-}Ax)^T (N\text{-}Ax) \tag{4}$$

And K is a large value assigned to the fitness function if the current solution x does not make the system fully observable; w_1 and w_2 are two user defined weights that ensure that the two parts of the fitness function F_1 and F_2 are comparable. The elements of the binary vector x are defined as follows:

$$x_i = \begin{cases} 1 \ \ if\, a\, PMU\, is\, place\, at\, bus\, i \\ 0 \ otherwise \end{cases} \tag{5}$$

The connectivity matrix of the system, A is given by:

$$A = \begin{cases} 1 \ if\, i = j \\ 1 \ if\, i\, and\, j\, are\, connected \\ 0 \ otherwise \end{cases} \tag{6}$$

The vector N is chosen to give the desired level of measurement redundancy at a bus. For example if all entries of N are set to 2, the level of measurement redundancy at each bus is 1.

IV. VSLI Estimation using an Echo State Network

Neural networks are known to be universal function approximators capable of learning the relationship between inputs and outputs of dynamic complex systems. The advantage of applying neural networks for power system identification is that once the neural network is trained, it is capable of estimating/predicting the right outputs for situations not seen previously without necessarily finding solutions of equations that define the dynamics of the system. In general prediction of outputs using neural networks is less computationally intensive and therefore takes much less computation time when compared to traditional methods of analysis such as power flow computation that involves solving systems of non-linear equations iteratively.

In this study, an echo state network has been used to estimate the voltage stability load index in a power system. The inputs used are measurements of voltage magnitudes and phase angles obtained using PMU models. PMUs are placed in the system at load buses that ensure the system is fully observable so that values of voltage magnitudes and phase angles are obtained from the minimum set of PMUs.

A. Echo State Network

The neural network used for estimating VSLI is a kind of recurrent neural network known as the echo state network. The ESN used to estimate VSLI on the IEEE 14 bus test system is shown in Fig. 4. The ESN consists of six input neurons, 100 neurons internal neurons and eleven output neurons. Where the inputs are PMU measurements of voltage magnitude and phase angle at each bus equipped with a PMU. An ESN consisting of a dynamic reservoir with input, internal and feedback weight matrices (W^{in} and W) respectively with the echo state property was developed following the steps described in [9]. The ESN outputs are the estimated values of VSLI at each load bus. The input and output layer neurons have linear functions; while dynamic reservoir neurons have sigmoid functions.

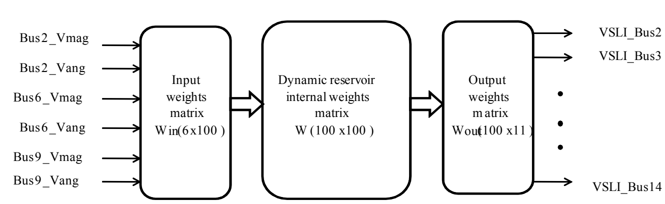

Fig. 4. Structure of ESN for VSLI estimation.

The untrained network was driven with the training data set consisting of PMU measurements of voltage magnitudes and phase angles at each bus equipped with a PMU. The target outputs are calculated VSLI at all load buses. The ESN internal states were calculated using (7).

$$x(n+1) = f(u(n+1)W^{in} + x(n)W) \tag{7}$$

The ESN internal state **x**(n) and output **d**(n) were collected into a state teacher matrix **M** and output teacher matrix, T, respectively. The first few terms of **T** are neglected allowing for washout since the initial state **x**(0) and output **d**(0) are not defined and are both set to zero. Output weights, $\mathbf{W}^{\mathbf{out}}$ are computed by multiplying the pseudoinverse of the matrix **M** by the matrix **T** as given below:

$$W^{out} = M^{-1}T \tag{8}$$

Finally utilizing the trained output weight matrix and the dynamic reservoir, the ESN network is used for estimating the desired output when presented with input information using (7) and (9):

$$y(n+1) = f(u(n+1)W^{out}, x(n+1), y(n)) \tag{9}$$

C. *Development of the ESN for VSLI Estimation*

The process followed in the development of an ESN involved two phases shown in a flowchart in Fig. 5. In the development phase, training data was obtained varying the load factor, λ at each load bus in the range 0 to 0.7. In order to ensure variability of load, random values of load factor were used at each bus for each training set. Generator output was increased in proportion to the total system load increase. The load at each bus, P_{Li} and generator output P_{Gi} are given by (10) and (11) respectively, and P_{L0i} and P_{G0i} are base case values of real power at load buses and generator buses, respectively.

$$P_{Li} = P_{L0i} * (1 + \lambda_i) \tag{10}$$

$$P_{Gi} = P_{G0i} * (1 + \lambda_i) \tag{11}$$

PMU measurements of voltage magnitudes and phase angles are obtained as input data for VSLI estimation for each load factor. A total of 250 inputs and output training data sets were obtained.

In the second phase, the operation phase, the trained ESN was applied in the estimation of voltage stability load index at each load bus. Fifty sets of input output patterns were used to test the accuracy of the ESN in estimating the voltage stability load index.

The proposed method for optimal PMU placement considering islanding was applied on the IEEE 14-bus system shown in Fig. 6.

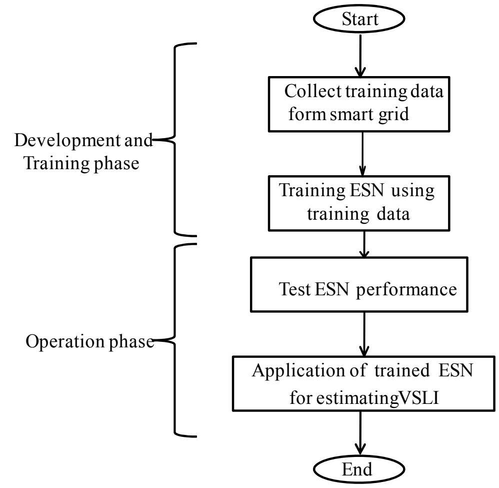

Fig. 5. Development and operation phases for VSLI estimation.

V. Real-Time Simulation Results

Tables I - III show results for optimal PMU placement considering islanding conditions. The 14 bus system has been split into two islands and the optimal locations of PMUs for complete observability both during normal operating conditions and during islanding conditions has been obtained. Results show that for the same level of redundancy, the system utilizes more PMUs when islanding operating conditions are considered. A total of four PMUs is required in order to ensure full observability during both normal operating conditions and during islanding conditions, whereas only three PMUs are need if islanding is not considered.

Three cases for VSLI estimation were considered for the IEEE 14 bus system. The first one being the normal operating condition, and the second and third cases were two N-1 contingencies involving the loss of lines 2-3 and line 2-4 respectively.

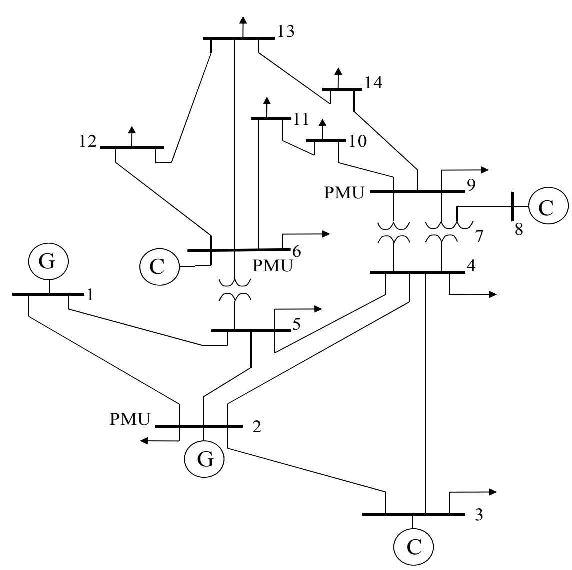

Fig. 6. IEEE 14-bus test system.

TABLE I
IEEE 14-BUS SYSTEM SPLIT INTO TWO ISLANDS

No.	Buses	Lines opened
1	1,5,6,11,12,13	1-2,2-5,4-5,11-10,13-14
2	2,3,4,7,8,9,10,14	

TABLE II
OPTIMAL PMU LOCATIONS FOR IEEE 14-BUS SYSTEM

System Configuration	Optimal PMU Locations
Normal operating conditions	2,6,9
Split into two islands	1,2,6,9

Performance of the ESN for the estimation of VSLI is shown in Tables IV and V. The ESN training is implemented in a one step calculation and carried out on a desktop computer (processor speed 3.33GHz). The tables show that the training time was less than 0.1 second for an ESN.

TABLE III
OPTIMAL NUMBER OF PMUS AND REDUNDANCY FOR THE IEEE 14-BUS SYSTEM

System Configuration	Optimal Number of PMUs	Number of Times each bus is observed
Normal operating conditions	3	1,1,1,1,2,1,1,1,1,1,1,1,2,1
Split into three islands	4	1,1,1,2,2,1,1,1,1,1,1,1,1,1

TABLE IV
ESN PERFORMANCE ON TRAINING

	ESN		
	Number of Weights	Training Time (sec)	Mean square Error (%)
IEEE 14 bus system	1100	0.067	4.31×10^{-5}

TABLE V
ESN PERFORMANCE ON INDIVIDUAL LOAD BUSES

Bus No.	Mean Square Error
2	4.27×10^{-5}
3	5.36×10^{-6}
4	3.21×10^{-4}
5	1.11×10^{-5}
6	1.91×10^{-5}
9	1.18×10^{-6}
10	9.38×10^{-6}
11	1.2×10^{-5}
12	4.27×10^{-5}
13	4.44×10^{-5}
14	4.3×10^{-5}
Ave MSE	4.31×10^{-5}
Max MSE	3.21×10^{-4}
Min MSE	1.18×10^{-6}
Std dev	9.3374×10^{-5}

The accuracy of the trained ESN in estimating the VSLI is shown in Table V. The table shows that on average the mean square error on testing for the ESN was 4.31×10^{-5}.

System voltage profiles shown in Fig. 7 show that as load factor is increased, voltage at load buses declined. Bus 14 has the lowest voltage profile while bus 2 is the strongest bus with a higher voltage profile.

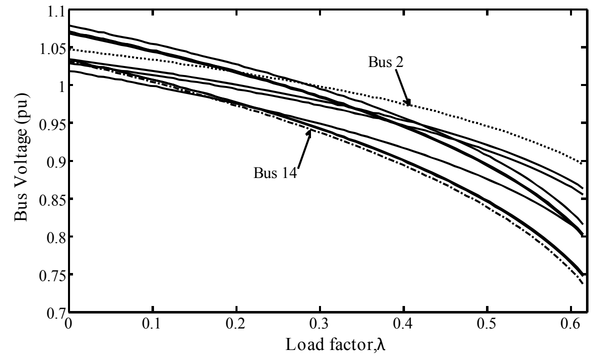

Fig.7. Bus voltage profiles for the IEEE 14 bus system.

Graphs of VSLI for the IEEE 14 bus test system are shown in Figs. 8 – 10. Fig. 8 shows that bus 14 has the highest value of VSLI and the value approaches one as the load factor approaches 0.62. This result is in agreement with the result found using P-V curve analysis in [21].

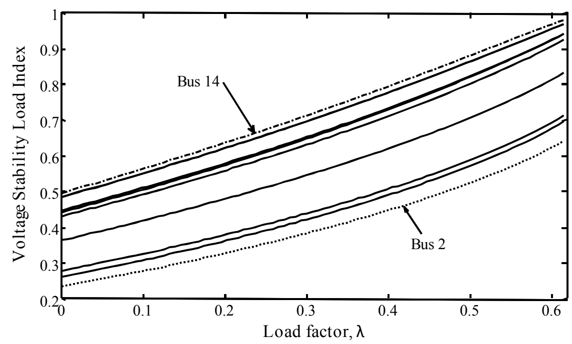

Fig. 8. VSLI for the IEEE 14 bus system.

Finally, estimated VSLI of the system for the two N-1 contingencies are shown in Figs. 9 and 10. In Fig. 9, the system approaches voltage instability at load factor of nearly 0.45 while in Fig. 10 the system approaches voltage instability at the load factor of nearly 0.5. This shows that the system has a lower voltage stability margin following the loss of line 2-3 than when line 2-4 was lost.

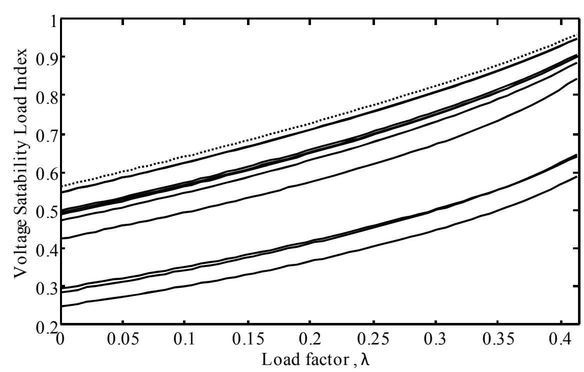

Fig. 9. Estimated VSLI for the IEEE 14 bus system with the loss of line 2-3.

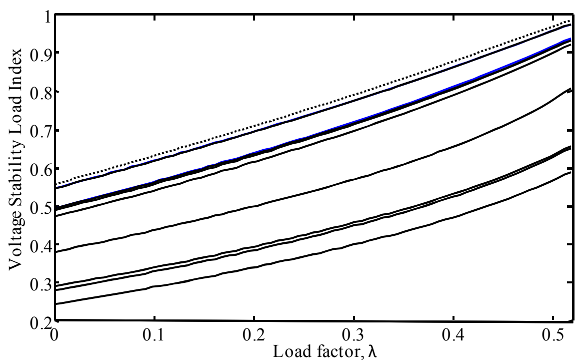

Fig. 10. Estimated VSLI for the IEEE 14 bus system with the loss of line 2-4.

VI. Conclusion

A methodology for estimation of voltage stability load index in a power system based on an echo-state network using PMU measurements has been presented in this paper. In addition, optimal PMU placement in a power system considering islanding operation conditions using genetic algorithm is carried out to ensure that the islands are also observable. Performance of the ESN for estimating VSLI has been evaluated and the results show high accuracy in the VSLI estimation under normal and disturbance conditions.

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