# Heuristic Algorithms for Solving Convex and Nonconvex Economic Dispatch

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**Abstract— Economic dispatch (ED) is a power system optimization problem and its objective is to reduce the total generation cost of units while satisfying constraints. The presence( r)-4n(s)-e(e)-e(t**

**compared as a way of demonstrating and validating the heuristic algorithms in solving this complex and challenging power system problem characterized by practical and nonconvex generator constraints.**

*Index Terms—***Differential evolution, economic cost function, economic dispatch, generation cost, genetic algorithm, particle swarm optimization, prohibited operating zones, ramp-rate limits.**

## NOMENCLATURE

| | |
|---|---|
| $a_i, b_i$ & $c_i$ | Fuel cost coefficients for unit $i$ |
| $\beta_{gb}$ | Global best strategic learning parameter |
| $c_1$ & $c_2$ | Cognitive and social acceleration constants respectively |
| $d$ | Particle's dimension |
| $C_R$ | Crossover constant |
| $S_i$ | Generator status indicator |
| $F$ | Scaling factor for mutation |
| $fit$ | Fitness |
| $i$ | Index of running generating units |
| $Iter$ & $Iter_{max}$ | Current and maximum iteration numbers respectively |
| $t$ | Continuous time step |
| $K_{pb}$ | Penalty factor coefficient for ED real power balance constraint |
| $l$ | $l$th particle |
| $np$ | Number of population in a generation |
| $N$ | Total number of online generating units |
| $P_{gd}$ | Swarm's best position for dimension $d$ |

| | |
|---|---|
| $P_{lbd}$ | $l$th particle best position for dimension $d$ |
| $P_{ld}$ | Position vector of the particle $l$ in dimension $d$ |
| $P_i$ | Generating capacity of unit $i$ |
| $P_{loss}$ | System loss |
| $P^D$ | Total real load demand |
| $r, rand, rand_1$ & $rand_2$ | Random numbers with uniform distribution in the range of [0, 1] |
| $randn$ | Gaussian distributed random number with a zero mean and a variance of 1 |
| $U^o$ | Initial random population |
| $U_1, U_2$ and $U_3$ | Randomly selected parents population vectors |
| $U_3'$, $U_i$ & $U_t$ | Offspring, primary array and trial vectors respectively |
| $V_{ld}$ | $l$th particle velocity in dimension $d$ |
| $w, w_{max}$ and $w_{min}$ | Current, final and initial inertia weights respectively |

## I. INTRODUCTION

ECONOMIC dispatch (ED) is one of the important optimization problems in power systems that has the objective of dividing the power demand among the online generators economically while satisfying various constraints [1]. Modern power system is experiencing increased demand for electricity with related expansions in system size, which has resulted in higher number of generators and lower reserve margins making the ED problem more challenging and complicated [1]. Conventional dispatch algorithms employ Lagrangian multipliers and require monotonically increasing cost curves. Unfortunately, the input-output characteristics of modern generating units are inherently highly nonlinear due to valve-point loading effects, ramp-rate limits, prohibited operating zones and so on, which tend to generate multiple local minima points in the cost function [2], [3]. Classical dispatch algorithms require that these characteristics be approximated, however such approximations may lead to suboptimal operation of the generator and results in heavy revenue losses.

The basic ED considers the power balance constraint apart from the generating capacity limits. However, a practical ED must take ramp-rate limits, prohibited operating zones, valve-point effects, and multi-fuel options into consideration to provide the completeness for the ED formulation. The resulting ED is a nonconvex optimization problem, which is a

challenging one and cannot be solved by traditional methods. Some factors that influence ED of the system are operating efficiency of generating units, fuel and operating costs, and transmission losses [1], [2]. The ED problems are in general nonconvex optimization problems with many local minima. Numerous classical techniques such as Lagrange based algorithms, linear programming (LP), non-linear programming (NLP) and quadratic programming (QP)and dynamic programming (DP) algorithms have been reported in the literature [4], [5].

Optimization problems in power system including ED have complex and nonlinear characteristics with stringent equality and inequality constraints to be satisfied. Different optimization techniques applied so far to solving these problems can be classified according to the type of the search space and/or the objective function [4]-[6]. Depending on the problem formulation, the objective function could be minimization of the unit generation costs, maintenance costs or some predefined reliability risks subject to some constraints resulting in nonlinear optimization as proposed in [4]-[6]. Solving such nonlinear optimization problems for most cases may not be feasible because their numerical solutions require extensive computational efforts, which increase exponentially with the problem complexities. Even though deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters [6], [7].

This paper presents three heuristic algorithms, namely, genetic algorithm (GA), differential evolution (DE) and modified particle swarm optimization (MPSO), used to solve these highly challenging ED problems of two test systems whose generating units are characterized by convex and nonconvex operational features. Solving this practical optimization problem leads to a minimized total generation cost of operating the two respective power systems in the presence of generator constraints.

The primary contributions of this paper are:

- Application of GA, DE and MPSO algorithms to solving the ED problem of two test systems with convex and nonconvex fuel cost functions.
- Comparison of performances of GA, DE and MPSO based ED algorithms for minimization of economic costs objective function.
- Application of the heuristic algorithms to solving the nonconvex ED problems where the classical Lagrange based algorithm cannot be directly applied.
- Solving GA, DE and MPSO based ED problems considering other practical generator constraints, namely, load balance, generator ramp-rate limits, prohibited operating zone and spinning reserve.

## II. PROBLEM FORMULATION OF ED

The ED problem is to find the optimal combination of power generations that minimizes the total generation cost while satisfying some constraints. The ED problem is commonly formulated as costs optimization problems, with the aim of minimizing the total generation cost of the power system but still satisfying equality and inequality constraints.

The objective here is to minimize the economic cost function expressed as second order function of each unit output $P_i$ as shown in (2) subject to satisfying practical generator constraints.

The prohibited operating zones in the input-output performance curve for a typical thermal unit can be due to vibrations of shaft bearing caused by a steam valve or the associated auxiliary equipment such as boilers, feed pumps etc [1], [8]. Practically, the shape of input-output curve in the neighborhood of a prohibited zone is difficult to determine by actual performance testing or from operational records. Hence in actual operation, the best economy is achieved by avoiding the operation in these areas [1], [8]. Cost function or the constraints can be modeled to include the prohibited operating zones according to the description shown in Fig. 1.
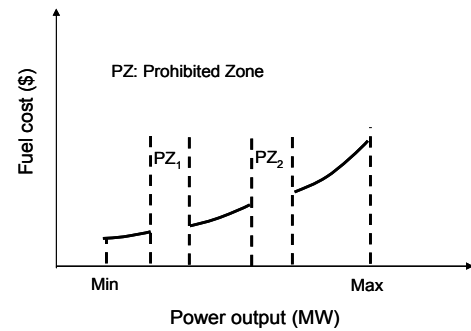


Fig. 1. Example of cost function with two prohibited operating zones

It is commonly assumed that the generation output can be adjusted instantaneously. Although this assumption is useful because it simplifies the problem, it however does not reflect the actual operating process of the generating unit. The operating range for online generating units is practically restricted by their ramp-rate limits [1], [2], [8].

The inclusion of the prohibited zones, ramp-rate limits and other practical constraints results in nonconvex ED of generating units. The practical ED considering generator nonlinearities outlined above are modeled in this paper.

Define

$$S_i = \begin{cases} 1 & \text{if unit } i \text{ is offline} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The objective function can be expressed as

$$\min F = \sum_{i=1}^{N} \left\{ \left( a_i + b_i P_i + c_i P_i^2 \right) \left( 1 - S_i \right) \right\} \quad (2)$$

subject to the following ED constraints:

- *Load balance*
  The generated power from all the running units must satisfy the load demand and the system losses given by (3).

$$\sum_{i=1}^{N} P_i = P^D + P_{loss} \quad (3)$$

To calculate system losses, algorithms based on penalty factors and constant loss formula coefficients or the B-coefficients are in use [1], [8]. System losses expression based on the B-coefficients is used in this paper and is given by (4).

$$P_{loss} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{oi} P_i + B_{oo} \tag{4}$$

- *Generator ramp-rate limits*

The power output of a practical generator cannot be adjusted instantaneously without limits. The operating range of all online units is restricted by their ramp-rate limits during each dispatch period. Therefore, the subsequent dispatch output of a generator should be limited between the constraints of up and down ramp-rates [1], [2], [8], hence the generator operating limits given by (5) are modified according to (6).

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{5}$$

$$\max(P_i^{\min}, P_i^{pre} - DR_i) \le P_i \le \min(P_i^{\max}, P_i^{pre} + UR_i) \tag{6}$$

- *Prohibited operating zone*

Each generator has its generation capacity, which cannot be exceeded at any time. It is common for a typical thermal unit to have a steam valve in operation, or a vibration in a shaft bearing, which may result in interference and discontinuous input-output performance-curve sections [1], known as the prohibited zones, as shown in Fig. 1. Practically, adjusting the power output of a unit must avoid all capacity limits and unit operations in prohibited zones [8]. The acceptable operating zones of a generating unit can be formulated as shown in (7).

$$\left. \begin{aligned} P_i^{\min} &\le P_i \le P_{i,1}^{lower} \\ P_{i,j-1}^{upper} &\le P_i \le P_{i,j}^{lower} \quad j = 2,3,..., pz_i \\ P_{i,pz_i}^{upper} &\le P_i \le P_i^{\max} \end{aligned} \right\} \tag{7}$$

- *Spinning reserve constraint*

Sufficient spinning reserve is required from all running units to maximize and maintain system reliability.

$$\sum_{i=1}^{N} P_i^{\max} \ge P^D + R \tag{8}$$

### III. ECONOMIC DISPATCH PROBLEM

The load demand is distributed among the running units in ED. The generation output of each unit should lie between the minimum and maximum power limits for good ED [1]. While minimizing the total generation cost, the total generation from running units should be equal to the total system demand plus the transmission network loss. The ED consists of finding the optimum operating policy and distribution of power among the running units while satisfying constraints (3-7) [1].

The evaluation function *f* (which is also called the fitness function in evolutionary algorithms), is defined for evaluating the fitness of each individual (or particle) in the generation (or swarm). The penalty function algorithm uses functions to penalize the objective function or the fitness of the individual (or particle) in proportion to the magnitude of the constraint violation [1]. The penalty function parameter is selected to distinguish between infeasible and feasible solutions.

In order to emphasize the 'best' solution and speed up convergence of the iterative procedure, the evaluation function *f* is defined according to (9) to minimize the fuel cost function given by (2) for a specified load demand $P^D$ while satisfying the system constraints.

$$f = F + K_{pb} \left( \sum_{i=1}^{N} P_i - P^D - P_{loss} \right)^2 \tag{9}$$

In order to limit the evaluation value of a potential solution within a feasible range, the generators' real output power operating limits constraint in (7) should be satisfied. If a potential solution satisfies this constraint, then it is a feasible solution and *f* has a relatively minimal evaluation value *f*. Otherwise, the *f* value of this potential solution is penalized.

*A. General Pseudo Code for GA, DE and MPSO based ED Algorithms*

*Step* 1: Specify the generation power limits, ramp-rate limits and prohibited operating zones of each unit.
*Step* 2: Set all parameters
*Step* 3: Select GA, DE or MPSO?
*Step* 4: Initialize randomly the chromosomes/ individuals/ particles in a generation/ iteration to lie within the specifications in *Step* 1.
*Step* 5: Evaluate the fitness values of all chromosomes/ individuals/ particles.
*Step* 6: Perform selection, crossover and mutation to determine best fit chromosome/ individual in a generation for GA or DE, or determine *pbest* and *gbest* of the particles' population, and perform mutation on *gbest* for MPSO.
*Step* 7: Update the velocity and position of each particle in MPSO.
*Step* 8: Determine the best chromosome/individual/global best particle.
*Step* 9: Terminate and print results if maximum generation/ iteration is reached. Otherwise, go to *Step* 4.

*B. GA Based Economic Dispatch*

Genetic algorithm is a search algorithm based on the modeling of natural genetics and natural selection [5]–[7]. The three main operators in used in GA are reproduction, crossover and mutation.

Implementation of a problem in a GA starts from the parameter encoding (that is, the representation of the problem). The encoding must be carefully designed to utilize the GA's ability to efficiently transfer information between chromosome strings and objective function of the problem. The GA for this ED problem is encoded by grouping the on-

line units for ED according to Fig. 2. Each chromosome consists of maximum number of units as genes, with each gene encoded as 16 bits. As shown in Fig. 2, each generating unit power output is concatenated and encoded in a binary based string normalized over its operating range. In this paper, each generating unit string is assigned by 16 bits, thus a string individual has $16 \times N$ bits [7].

To obtain the actual power output of each generating unit for fitness evaluation, each string is decoded to the decimal value using (9) [7].
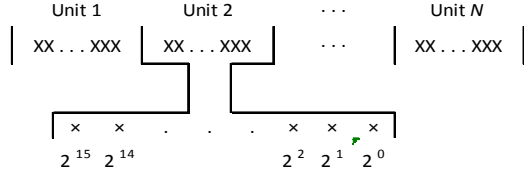


Fig. 2 $16 \times N$ bits concatenated binary coding scheme of generating units

Evaluation of a chromosome is achieved by decoding the encoded chromosome string using (10) and then computing its fitness.

$$P_i = P_i^{\min} + \frac{D_i \times \left(P_i^{\max} - P_i^{\min}\right)}{2^{16} - 1} \quad \text{for } i = 1, 2, \ldots, N \tag{10}$$

Where (10) ensures that the generating units' output power $P_i$ lies within the units' operating real power limits.

### C. DE Based Economic Dispatch

Differential evolution is an optimization algorithm that solves real-valued problems based on the principles of natural evolution [5]. Like other evolutionary algorithms, DE also relies on initial random population generation, which is then improved using selection, mutation, and crossover repeated through generations until the convergence criterion is met [5].

Although the canonical form of differential evolution solves optimization problems over continuous spaces, minor adjustments to the code allow DE to solve mixed integer optimization problems [5]. This is achieved with the use of operator that rounds the variable to the nearest integer value, when the value lies between two integers.

An initial population composed of vectors $U_i^o$, $i = 1, 2, \ldots np$, is randomly generated within the parameter space. In each generation, $np$ competitions are held to determine the composition of the next generation. Every pair of randomly chosen vectors $U_1$ and $U_2$ defines a vector differential: $(U_1 - U_2)$. Their weighted differential is used to perturb another randomly chosen vector $U_3$ according to (11) given by:

$$U_3^{'} = U_3 + F * (U_1 - U_2) \tag{11}$$

Typically $(0 \leq F \leq 1.0)$ and it controls the speed and robustness of the search; a lower value increases the rate of convergence but also the risk of being stuck at the local optimum. The crossover is a complimentary process for DE. It aims at reinforcing the prior successes by generating the offspring vectors. In every generation, each primary array vector $U_i$, is targeted for crossover with a vector like $U_3^{'}$ to produce a trial vector $U_t$ according to (12).

$$U_t = \begin{cases} U_3^{'} & \text{if } rand < C_R \\ U_i & \text{otherwise} \end{cases} \tag{12}$$

The newly created vector is evaluated by the objective function and the corresponding value is compared with the target vector. The best fit vector is kept for the next generation as given by (13). The best parameter vector is evaluated for every generation in order to track the progress made throughout the minimization process; thus making the DE elitist algorithm.

$$U_i(t+1) = \begin{cases} U_i(t) & \text{if } fit(U_i(t)) \leq fit(offspring\ (t)) \\ offspring & \text{otherwise} \end{cases} \tag{13}$$

The best parameter vector evaluated for the latest generation produce the economic dispatch generation with the minimum generation cost [8], [9].

### D. MPSO Based Economic Dispatch

The modified PSO is a combination of PSO and an evolutionary strategy enhancing the method to perform optimal search under complex environments [10]

Let $X$ and $V$ denote a particle coordinates (position) and its corresponding flight speed (velocity) in a search space, respectively. Therefore, the $l$th particle is represented as $X_{ld} = (X_{l1}, X_{l2}, \ldots, X_{lN})$ in the $d$-dimensional space. The best previous position of the $l$th particle is recorded and represented as $P_{lbd} = (P_{lb1}, P_{lb2}, \ldots, P_{lbN})$. The index of the best particle among all the particles in the group is represented by the $P_{gd}$. The rate of the velocity for particle $l$th is represented as $V_{ld} = (V_{l1}, V_{l2}, \ldots, V_{lN})$. In this version of PSO, the velocity is limited to a certain range $[-V_{max}, V_{max}]$ such that $V_{ld}$ always lies in that range [10]. The new velocity and position for each particle $i$ in dimension $d$ is determined according to the velocity and position update equations given by (14) and (15), while the inertia weight is updated according to (16).

$$V_{ld}(t) = w \cdot V_{ld}(t-1) + c_1 \cdot rand_1 \cdot (P_{lbd}(t-1) - X_{ld}(t-1))$$
$$+ c_2 \cdot rand_2 \cdot (P_{gb}^{*}(t-1) - X_{ld}(t-1)) \tag{14}$$

$$X_{ld}(t) = X_{ld}(t-1) + V_{ld}(t) \tag{15}$$

$$w = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{iter_{\max}}\right) \times iter \tag{16}$$

The MPSO is a combination of PSO and an evolutionary strategy enhancing the algorithm to perform optimal search under complex environments [10]. This version of MPSO is a variant of the original formulation of the continuous particle swarm optimization (CPSO) to solve continuous optimization problems such as the ED problem considered in this paper. Supposing $P_{gd}$ is the swarm's global best particle chosen with a random number less than a predefined mutation rate (for $0 <$

mutation rate < 0.2) then the mutation result of this particle is given by (17).

$$P_{gd}^*(t-1) = P_{gd}(t-1) + ceil(randn \times P_{gd}(t-1)/\beta_{gb}) \quad (17)$$

$d$=1, 2, …, N

where $\beta_{gb}$ is global best strategic learning parameter for mutation that can be either dynamically changing or fixed and controls the mutation process introduced in this MPSO algorithm. The main goal is to increase the diversity of the population by preventing the particles from moving too close to each other, thus converging prematurely to local optima. This in eventually improves the CPSO's search performance.

The control or the decision variables for the ED problems are real power generations, and are therefore used to form the swarm. The real power outputs of all on-line generators are represented as the positions of the particles in the swarm [11]–[14]. For $N$ on-line generators, the particle position is represented as a vector of length $N$. If there are $N_{par}$ particles in the swarm, the $l$th particle in the swarm can be represented as a matrix shown in (18).

$$P_{ld} = [P_{l1}, P_{l2},...,P_{li},...,P_{lN}] \quad (18)$$

$d$=1, 2, …, i, …, N

Where $P_{ld}$ is the position vector of the particle $l$ in dimension $d$ and represents a potential solution to the optimization problem. The element $P_{li}$ of the $P_{ld}$ vector is the $l$th position component of particle $l$, and it represents the real power generation of on-line generator $i$ of the possible solution.

Each element of the swarm matrix is initialized randomly within the effective real power operating limits. The initialization is either based on (5) for generators without ramp-rate limits, or on (6) for generators with ramp-rate limits which is the case considered in this paper.

The $i$th dimension of the $l$th particle is assigned a value of $P_{li}$ given by (19) to satisfy the constraint given by (6).

$$P_{li} = P_{li}^{min} + r*(P_{li}^{max} - P_{li}^{min}) \quad (19)$$

The fitness values obtained from (9) for the initial particles of the swarm are set as the initial *pbest* values of the particles. The best value among all the *pbest* values becomes the *gbest*. Mutation operator is introduced into the algorithm using (17).

The new velocity is computed using (14). To control excessive roaming of particles, the velocity is limited to a certain range [- $V_{max}$, $V_{max}$] such that $V_{ld}$ always lies in that range. The maximum velocity is limited to between 10% - 20% of the dynamic range of the variable on each dimension.

The swarm is updated by updating the particle's position vector using (15). The *pbest* and *gbest* values are subsequently updated.

## IV. CASE STUDIES, NUMERICAL RESULTS AND ANALYSIS

### A. Case Studies

Two case studies are presented in this section of the paper for solving the convex and nonconvex static economic dispatch problems using heuristic algorithms.

*Case I*

The test system for this case consists of 6 thermal units, including the generation limits, fuel cost coefficients, ramp-rates limits and prohibited operating zones of these thermal units, 26 buses and 46 transmission lines [14]-[16]. The system real power loss is also considered in this case. The loss coefficients on 100 MVA base capacity are presented in (20 - 22). The total load demand is 1263. There is 207MW of total spinning reserve accruable from the 6 thermal units, amounting to 14.08% of total generation thus satisfying the constraint in (8).

Using the 6-unit data, the total generation cost resulting from the online units can be evaluated based on their economic dispatch generation after minimization of the objective function in (2) subject to the constraints (3-8).

$$B_{ij} = \begin{bmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{bmatrix} \quad (20)$$

$$B_{oi} = 1.0e^{-03} * [-0.3908 \quad -0.1297 \quad 0.7047 \quad 0.0591 \quad 0.2161 \quad -0.6635] \quad (21)$$

$$B_{oo} = 0.0056 \quad (22)$$

*Case II*

The test data for this case is taken from a real power system consisting of 19 generating units characterized by convex fuel cost coefficients, taken from two industrial parks located in Bintan and Batam in Indonesia [17]. The total generation cost is evaluated as described in Case I.

### B. Numerical Results and Analysis

All numerical results are obtained based on programs developed using Matlab environment and ran on PC with 2.2GHz CPU speed and 1.5GB of RAM. Results of the two cases are presented below.

The following parameters are used by the three heuristic algorithms:

*GA*

Generation size of 50 chromosomes (or individuals), maximum generation number of 500 and 300 for Cases I and II respectively, crossover rate of 0.7, linearly decreasing mutation rate (from 0.7 to 0.1), $\beta_{gb}$ of 2 and $K_{pb}$ is empirically tuned to value of 1000 for the two cases.

*DE*

Generation size of 50 individuals, maximum generation number of 500 and 300 for Cases I and II respectively, scaling factor for mutation is 0.5, crossover rate of 0.8, $\beta_{gb}$ of 2 and $K_{pb}$ is empirically tuned to value of 1000 for the two cases.

*MPSO*

Population size of 30, $w_{min}$ and $w_{max}$ of 0.4 and 0.9 respectively, $c_1$ and $c_2$ of 2 each, $V_{max}$ is 20% of the dynamic range of the variable on each dimension, mutation rate of 0.15, $Iter_{max}$ of 500 and 300 for Cases I and II respectively, $\beta_{gb}$ of 2 and $K_{pb}$ is empirically tuned to value of 1000 for the two cases.

*Case I*

Table I shows the ED schedules generated by each of the 6 thermal units, total generation, total power loss and total generation costs produced by GA, DE and MPSO based ED algorithms. The table shows the amount of power generation economically dispatched to meet the load demand of 1263 MW while satisfying constraints (3-8). Total minimum generation costs in meeting load demand, as produced by GA, DE and MPSO based ED algorithms are $15450, $15446 and $15444 respectively, while the real power losses produced are 12.3434MW, 12.4900MW and 12.0141MW respectively. Dispatch results from MPSO based ED algorithm is seen to result in the least real power loss compared to the GA and DE based ED algorithms. The MPSO algorithm also generated the most economical and minimal total generation cost compared with the other two algorithms. These results demonstrate MPSO based ED algorithm's better ability in solving the ED problem compared with the GA and DE based ED algorithms for units characterized by ramp-rate limits and prohibited operating zones. The GA algorithm on the other hand generated lower real power loss compared with the DE algorithm.

### TABLE I
POWER GENERATION AND GENERATION COSTS DATA FOR THE 6-UNIT TEST SYSTEM

| Generating units | Algorithms | | | | | | | |
| | 100 Trials (Proposed) | | | 50 Trials [14] | | 100 Trials [15] | 50 Trials [16] | |
| | GA | DE | MPSO | GA [14] | PSO [14] | NPSO-LRS [15] | PSO [16] | MPSO [16] |
|---|---|---|---|---|---|---|---|---|
| $P_1$ | 439.7774 | 445.5862 | 438.9866 | 474.8066 | 447.4970 | 446.9600 | 451.2741 | 444.8882 |
| $P_2$ | 179.6234 | 187.4714 | 162.9011 | 178.6363 | 173.3221 | 173.3944 | 162.4633 | 168.1455 |
| $P_3$ | 261.5945 | 261.7543 | 267.0032 | 262.2089 | 263.4745 | 262.3436 | 262.6419 | 265.0000 |
| $P_4$ | 133.5927 | 130.3120 | 138.7787 | 134.2826 | 139.0594 | 139.5120 | 130.3146 | 129.4751 |
| $P_5$ | 151.3102 | 160.1349 | 158.3033 | 151.9039 | 165.4761 | 164.7089 | 173.8361 | 173.0299 |
| $P_6$ | 109.4452 | 90.2312 | 109.0412 | 74.1812 | 87.1280 | 89.0162 | 95.1188 | 95.0435 |
| Total generation over an hour (MW) | 1275.3434 | 1275.4900 | 1275.0141 | 1276.0300 | 1276.0100 | 1275.9351 | 1275.6488 | 1275.5823 |
| Load supplied over an hour (MW) | 1263.0000 | 1263.0000 | 1263.0000 | 1263.0000 | 1263.0000 | 1263.0000 | 1263.0000 | 1263.0000 |
| $P_{loss}$ over an hour (MW) | 12.3434 | 12.4900 | 12.0141 | 13.0217 | 12.9584 | 12.9361 | 12.6448 | 12.6411 |
| Total min. generation cost over an hour ($) | 15450 | 15446 | 15444 | 15459 | 15450 | 15450 | 15446 | 15444 |

Table II shows the statistical comparison of the generation costs among all the ED algorithms considered in this paper. The table shows MPSO performing better than the GA and DE based ED algorithms in terms of the magnitude of variations in minimum generation costs and standard deviations. The results presented in Tables I and II shows a reduction in both the total generation costs and transmission losses produced by GA, DE and MPSO in this paper compared with the results

presented in [14]–[16] on the same test system. The significant power loss reductions with the MPSO presented in this paper have the benefits of energy saving, fuel cost reduction, and also emission of generated-unused power is reduced. The loss reductions are reflections of improvements in the performances of the algorithms presented in this paper.

Table III shows the statistical comparison of computation efficiency for all the three ED algorithms considered in this paper. The MDPSO based ED algorithm demonstrates faster computation time in locating minimum generation cost compared with GA and DE based ED algorithms as numerically shown in Table III for various numbers of maximum generations/iterations. The GA however, converged to global solutions at faster rate than the DE under similar operating conditions as seen in Table III.

### TABLE II
STATISTICAL COMPARISON OF GENERATION COSTS FOR THE 6-UNIT TEST SYSTEM

| Algorithms | | Total generation cost over an hour | | | |
| | | Min. ($) | Max. ($) | Ave. ($) | Std |
|---|---|---|---|---|---|
| 100 Trials (Proposed) | GA | 15450 | 15498 | 15478 | 25.2104 |
| | DE | 15446 | 15501 | 15483 | 28.3469 |
| | MPSO | 15444 | 15496 | 15457 | 24.8425 |
| 50 Trials [14] | GA [14] | 15459 | 15524 | 15469 | - |
| | PSO [14] | 15450 | 15492 | 15454 | - |
| 100 Trials [15] | NPSO-LRS [15] | 15450 | - | - | - |
| 50 Trials [16] | PSO [16] | 15446 | 15538 | 15477 | - |
| | MPSO [16] | 15444 | 15504 | 15460 | - |

### TABLE III
STATISTICAL COMPARISON OF COMPUTATION EFFICIENCY FOR THE 6-UNIT TEST SYSTEM

| Description | Algorithms | Generations/Iterations | | | | |
| | | 100 | 500 | 1000 | 3000 | 5000 |
|---|---|---|---|---|---|---|
| Total CPU time (sec) | GA | 2.51 | 11.58 | 23.88 | 84.85 | 175.32 |
| | DE | 9.19 | 44.71 | 92.11 | 284.96 | 510.68 |
| | MPSO | 0.59 | 1.63 | 3.05 | 8.55 | 14.29 |

Fig. 3 shows the minimum generation costs convergence performance for the GA, DE and MPSO based ED algorithms. The converged result conforms to the minimum generation costs presented in Tables I and II.
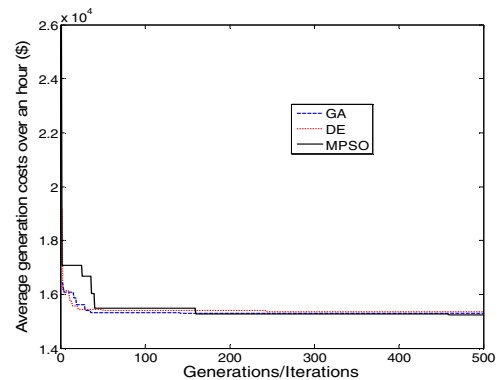


Fig. 3. Generation cost plots for the 6-unit test system

*Case II*

The ED schedules generated by each of the 19 online generating units using GA, DE and MPSO based ED algorithms are presented in Table IV. The table shows the

units' generation economically dispatched to balance up the supplied load as shown in Table IV while meeting the system constraints. The result shows that the DE and MPSO performed comparably well in terms of finding the most economical dispatch generation, total generation and minimum generation costs. Percent deviation errors in perfectly matching the supplied load of 70.2 MW produced by GA, DE and MPSO based ED algorithms are 0.062%, 0.066% and 0.00% respectively.

Table V shows the statistical comparison of the generation costs among all the ED algorithms considered. The table shows GA and MPSO algorithms performing fairly better than the DE algorithm in the statistical variation of the generation costs arrived at and its corresponding standard deviations.

Table VI shows the statistical comparison of computation efficiency for the GA, DE and MPSO based ED algorithms for the 19 units test system for different maximum generations/iterations. The data in table VI are shown plotted in Fig. 5.

in Tables IV and V. The MPSO and the GA algorithms generated better minimum generation costs than the DE algorithm.

Fig. 5 presents plots for comparison of the computation efficiencies of the GA, DE and MPSO based ED algorithms for the 6 and 19 units test systems. For the two test systems, the MPSO shows faster computation efficiency compared with any of the other two algorithms. The DE presents slower computation efficiency compared with the GA algorithm, even as the problem dimension is increased.
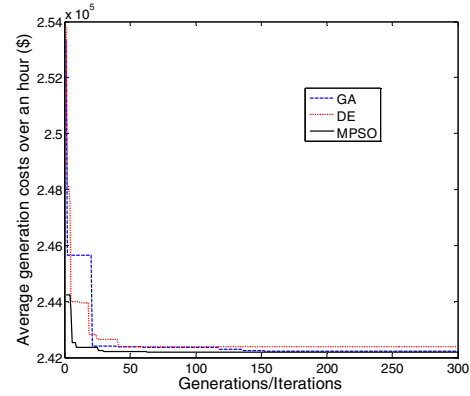


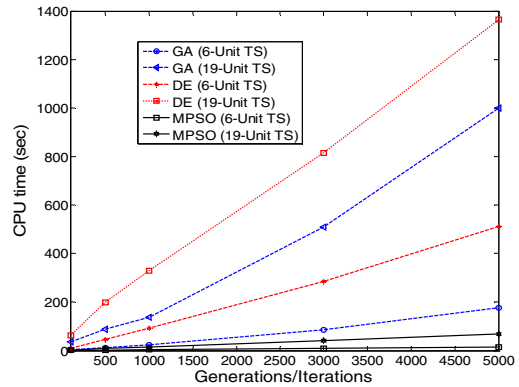Fig. 4. Generation cost plots for the 19-unit test system

TABLE IV
POWER GENERATION AND GENERATION COSTS DATA FOR THE
19-UNIT TEST SYSTEM

| Generating units | Generation (MW) | | | Generating units | Generation (MW) | | |
|---|---|---|---|---|---|---|---|
| | GA | DE | MPSO | | GA | DE | MPSO |
| $P_1$ | 5.8720 | 5.4398 | 4.1000 | $P_{14}$ | 7.1800 | 5.4582 | 6.0000 |
| $P_2$ | 1.7407 | 5.8829 | 3.6000 | $P_{15}$ | 2.5943 | 0.7223 | 2.1000 |
| $P_3$ | 5.8214 | 5.3230 | 6.1000 | $P_{16}$ | 1.0058 | 1.5431 | 1.1000 |
| $P_4$ | 4.5662 | 1.1908 | 3.1000 | $P_{17}$ | 1.7053 | 1.0481 | 2.1000 |
| $P_5$ | 4.9589 | 4.6090 | 3.1000 | $P_{18}$ | 0.8738 | 1.8333 | 2.1000 |
| $P_6$ | 4.7991 | 4.9791 | 6.1000 | $P_{19}$ | 5.3010 | 2.2615 | 2.1000 |
| $P_7$ | 2.8216 | 4.2533 | 6.1000 | Total gen. over a week (MW) | 70.2434 | 70.2461 | 70.2000 |
| $P_8$ | 1.9510 | 4.8229 | 5.1000 | | | | |
| $P_9$ | 3.8598 | 3.1180 | 2.1000 | Load supplied over a week (MW) | 70.2000 | 70.2000 | 70.2000 |
| $P_{10}$ | 5.9470 | 3.5836 | 3.4000 | | | | |
| $P_{11}$ | 2.6318 | 4.1133 | 2.4000 | Total generation cost over a week ($) | 242220 | 242240 | 242210 |
| $P_{12}$ | 1.7762 | 3.9459 | 4.4000 | | | | |
| $P_{13}$ | 4.8375 | 6.1180 | 5.1000 | | | | |

TABLE V
STATISTICAL COMPARISON OF GENERATION COSTS FOR THE
19-UNIT TEST SYSTEM

| Algorithms | Total generation cost over a week | | | |
|---|---|---|---|---|
| | Min. ($) | Max. ($) | Ave. ($) | Std |
| GA | 242220 | 242310 | 242300 | 28.4032 |
| DE | 242240 | 242340 | 242320 | 31.5620 |
| MPSO | 242210 | 242350 | 242280 | 40.0371 |

TABLE VI
STATISTICAL COMPARISON OF COMPUTATION EFFICIENCY
FOR THE 19-UNIT TEST SYSTEM

| Description | Algorithms | Generations/Iterations | | | | |
|---|---|---|---|---|---|---|
| | | 100 | 500 | 1000 | 3000 | 5000 |
| Total CPU time (sec) | GA | 34.94 | 87.63 | 136.52 | 509.10 | 999.32 |
| | DE | 63.73 | 199.24 | 330.16 | 815.24 | 1363.89 |
| | MPSO | 2.28 | 7.57 | 14.33 | 39.72 | 67.66 |

The convergence performance of the GA, DE and MPSO based ED algorithms for the 19 units test system is presented in Fig. 4. The figure shows the minimum generation costs convergence behavior for each of the three algorithms considered in this paper which simply corresponds to the best generation costs desired, and conforms to the result presented



Fig. 5. CPU time versus generations/iterations

## V. CONCLUSION

The economic dispatch (ED) problems for generators exhibiting practical and nonconvex characteristic behavior have been illustrated. Transmission line losses are incorporated to test the robustness of the heuristic algorithms. The heuristic algorithms have been compared and successfully applied in solving the ED problems of two practical power systems. The modified particle swarm optimization (MPSO) algorithm shows better performance in terms of the quality of results, loss reduction and computational efficiency in locating optimal solution when compared with the genetic algorithm (GA) and differential evolution (DE) under similar operating conditions. Increased problem dimensionality is seen not to have limitation on the algorithms' performances and in the quality of results obtained for the two power systems considered in this paper. The results offer good alternative for power system operational and planning activities in control areas desiring optimized energy management, generation costs curtailment and transmission loss reduction, in the face of

continuously increasing global fuel costs and the irreplaceable depletion of conventional raw fuel resources.

## VI. REFERENCES

[1] A. J. Wood, and B. F. Wollenberg, *Power generation operation and control,* John Wiley and Sons, New York, NY, 2004.

[2] C. Wang, and S. M. Shahidehpour, "Effects of ramp-rate limits on unit commitment and economic dispatch," *IEEE Transactions on Power Systems*, vol. 8, no. 3, pp. 1341-1349, August 1993.

[3] T. A. A. Victoire, and A. E. Jeyakumar, "Reserve constraint dynamic dispatch of units with valve-point effects," *IEEE Transactions on Power Systems*, vol. 20, no. 3, pp. 1273-1282, August 2005.

[4] H. T. Firma, and L. F. L. Legey, "Generation expansion: an iterative genetic algorithm approach," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 901-906, August 2002.

[5] A. P. Engelbrecht, *Fundamentals of computational swarm intelligence,* John Wiley & Sons Ltd, West Sussex P O 19 8SQ, England, 2005.

[6] I. G. Damousis, A. G. Bakirtzis, and P. S. Dokopolous, "Network-constrained economic dispatch using real-coded genetic algorithm," *IEEE Transactions on Power Systems*, vol. 18, no. 1, pp. 198-205, February 2003.

[7] J. Tippayachai, W. Ongsakul, and I. Ngamroo, "Parallel micro genetic algorithm for constrained economic dispatch," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 790-797, August 2002.

[8] A. Pereira-Neto, and O. R. Saavedra, "Efficient evolutionary strategy optimization procedure to solve the nonconvex economic dispatch problem with generator constraints," *IET Generation, Transmission and Distribution*, vol. 152, no. 5, pp. 653-660, September 2005.

[9] N. Sinha, R.Chakarabarti, and P. K. Chattopadhyay, "Evolutionary programming techniques for economic load dispatch," *IEEE Transactions on Evolutionary Computation*, vol. 7, no. 1, pp. 83-94, February 2003.

[10] Y. del Valle, G. K. Venayagamoorthy, S. Mohagheghi, J. Hernandez, and R. G. Harley, "Particle swarm optimization: Basic concepts, variants and applications in power system," *IEEE Transactions on Evolutionary Computation*, 12, (2), pp. 171-195, April 2008.

[11] C.-C. Kuo, "A novel coding scheme for practical economic dispatch by modified particle swarm approach," *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1825-1835, November 2008.

[12] J.-B. Park, K.-S. Lee, J.-R. Shin, and K. W. Lee, "A particle swarm optimization for economic dispatch with nonsmooth cost functions," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 34-42, February 2005.

[13] K. T. Chaturvedi, M. Pandit, and L. Srivastava, "Self-organizing hierarchical particle swarm optimization for nonconvex economic dispatch," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1079-1087, August 2008.

[14] Z.-L. Gaing, "Particle swarm optimization to solving the economic dispatch considering the generator constraints," *IEEE Transactions on Power Systems*, vol. 18, no. 3, pp. 1187-1195, August 2003. Closure to Discussion of 'particle swarm optimization to solving the economic dispatch considering the generator constraints," *IEEE Transactions on Power Systems*, vol. 19, no. 4, pp. 2122-2123, November 2004.

[15] A. I. Selvakumar, and K. Thanushkodi, "A new particle swarm optimization solution to nonconvex economic dispatch problems," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 42-51, February 2007.

[16] M. Neyestani, M. M. Farsangi, H. Nezamabadi-pour, and K. Y. Lee, "A modified particle swarm optimization for economic dispatch with nonsmooth cost functions," *in Proc. 2009 IFAC Symposium on Power Plants and Power systems Control (PP & PS)*,Tampere Hall, Finland.

[17] K. Y. Lee, and M. A. El-Sharkawi, *Modern heuristic optimization techniques: theory and applications to power systems.* IEEE Press, 445 Hoes Lane, Piscataway, New Jersey, 2008.